# Binomial Questions

**6** Plastic clothes pegs are made in various colours.

The number of red pegs may be modelled by a binomial distribution with parameter $p$ equal to 0.2.

The contents of packets of 50 pegs of mixed colours may be considered to be random samples.

(a) Determine the probability that a packet contains:

(i) less than or equal to 15 red pegs; *(2 marks)*

(ii) exactly 10 red pegs; *(2 marks)*

(iii) more than 5 but fewer than 15 red pegs. *(3 marks)*

(b) Sly, a student, claims to have counted the number of red pegs in each of 100 packets of 50 pegs. From his results the following values are calculated.

Mean number of red pegs per packet = 10.5

Variance of number of red pegs per packet = 20.41

Comment on the validity of Sly's claim. *(4 marks)*

---

**5** Kirk and Les regularly play each other at darts.

(a) The probability that Kirk wins any game is 0.3, and the outcome of each game is independent of the outcome of every other game.

Find the probability that, in a match of 15 games, Kirk wins:

(i) exactly 5 games; *(3 marks)*

(ii) fewer than half of the games; *(3 marks)*

(iii) more than 2 but fewer than 7 games. *(3 marks)*

---

**2** A hotel has 50 single rooms, 16 of which are on the ground floor. The hotel offers guests a choice of a full English breakfast, a continental breakfast or no breakfast. The probabilities of these choices being made are 0.45, 0.25 and 0.30 respectively. It may be assumed that the choice of breakfast is independent from guest to guest.

(a) On a particular morning there are 16 guests, each occupying a single room on the ground floor. Calculate the probability that exactly 5 of these guests require a full English breakfast. *(3 marks)*

(b) On a particular morning when there are 50 guests, each occupying a single room, determine the probability that:

(i) at most 12 of these guests require a continental breakfast; *(2 marks)*

(ii) more than 10 but fewer than 20 of these guests require no breakfast. *(3 marks)*

(c) When there are 40 guests, each occupying a single room, calculate the mean and the standard deviation for the number of these guests requiring breakfast. *(4 marks)*

---

**6** Each weekday, Monday to Friday, Trina catches a train from her local station. She claims that the probability that the train arrives on time at the station is 0.4, and that the train's arrival time is independent from day to day.

(a) Assuming her claims to be true, determine the probability that the train arrives on time at the station:

(i) on at most 3 days during a 2-week period (10 days); *(2 marks)*

(ii) on more than 10 days but fewer than 20 days during an 8-week period. *(3 marks)*

(b) (i) Assuming Trina's claims to be true, determine the mean and standard deviation for the number of times during a week (5 days) that the train arrives on time at the station. *(3 marks)*

(ii) Each week, for a period of 13 weeks, Trina's travelling colleague, Suzie, records the number of times that the train arrives on time at the station. Suzie's results are

2 2 4 1 2 3 3 2 2 0 3 2 0

Calculate the mean and standard deviation of these values. *(3 marks)*

(iii) Hence comment on the likely validity of Trina's claims. *(2 marks)*

---

# Binomial Answers

| | | | | |
|---|---|---|---|---|
| **6(a)(i)** | B(50, 0.2) | M1 | | Use of in (a) |
| | $P(R \leq 15) = 0.969$ to 0.97 | A1 | 2 | AWFW 0.9692 |
| **(ii)** | $P(R = 10) = P(R \leq 10) - P(R \leq 9)$ | | | Stated or implied |
| | **or** | M1 | | |
| | $P(R = 10) = \binom{50}{10}(0.2)^{10}(0.8)^{40}$ | | | Stated or implied |
| | $= 0.5836 - 0.4437 = 0.139$ to 0.141 | A1 | 2 | AWFW 0.1399 |
| **(iii)** | $P(5 < R < 15) =$ | | | |
| | $P(R \leq 14$ or $15) = 0.9393$ or 0.9692 | M1 | | Accept values to 3 dp |
| | minus $P(R \leq 5$ or $4) = 0.0480$ or 0.0185 | M1 | | Accept values to 3 dp |
| | $= 0.89$ to 0.893 | A1 | | AWFW 0.8913 |
| | **or** | | | |
| | B(50, 0.2) expressions stated for at least 3 of $5 \leq R \leq 15$ | (M1) | | Or implied by a correct answer |
| | Answer | (A2) | 3 | |

| | | | | |
|---|---|---|---|---|
| **(b)** | Mean, $\mu = np = 50 \times 0.2 = 10$ | B1 | | Either; CAO |
| | **or** | | | |
| | Estimate of $p$, $\hat{p} = 0.21$ | | | |
| | Variance, $\sigma^2 = np(1-p) = 10 \times 0.8 = 8$ | B1 | | CAO |
| | **Mean** or **Estimate of $p$** **is similar** to that expected | | | 10.5 and 10 or 0.21 and 0.2 |
| | but **Variance** (standard deviation) **is different** from that expected | B1 | | Either point<br>20.41 and 8 or 4.5 and 2.8 |
| | Reason to **doubt validity** of Sly's claim | B1 | 4 | Must be based on both 10 or 0.2 and 8 or on both 10 or 0.2 and 2.8 correctly |
| | **Total** | | **11** | |

| | | | | |
|---|---|---|---|---|
| **5(a)** | B(15, 0.3) | M1 | | use of in (a) |
| **(i)** | $P(K = 5) = P(K \leq 5) - P(K \leq 4)$ | | | |
| | $P(K = 5) = \binom{15}{5}(0.3)^5(0.7)^{10}$ | M1 | | may be implied |
| | $= 0.7216 - 0.5155 = 0.2055$ to $0.2065$ | A1 | 3 | AWFW (0.2061) |
| **(ii)** | (Fewer than) half $\Rightarrow$ 7 or 7½ or 8 | B1 | | stated or implied |
| | Thus require $P(K \leq 7$ or $< 8)$ | M1 | | used or implied by correct answer |
| | $= 0.9495$ to $0.9505$ | A1 | 3 | AWFW (0.9500) |
| **(iii)** | $P(2 < K < 7) =$ 0.8689 or 0.9500 | M1 | | |
| | minus 0.1268 or 0.2969 | M1 | | |
| | $= 0.7415$ to $0.7425$ | A1 | 3 | AWFW (0.7421) |
| | or | | | |
| | B(15, 0.3) expressions stated for at least 3 terms within $2 \leq K \leq 7$ | M1 | | or implied by a correct answer |
| | Answer | A2 | | |

---

| | | | | |
|---|---|---|---|---|
| **2(a)** | Use of binomial in (a), (b) or (c) | M1 | | Can be implied |
| | $P(E = 5) = \binom{16}{5}(p)^5(1-p)^{11}$ | M1 | | Allow $p = 0.45, 0.25, 0.30$ or $\frac{1}{3}$ |
| | $= 0.112$ | A1 | 3 | AWRT (0.1123) |
| **(b)(i)** | B(50, 0.25) | B1 | | Used; can be implied |
| | $P(C \leq 12) = 0.511$ | B1 | 2 | AWRT (0.5110) |
| **(ii)** | $P(10 < B' < 20) =$ 0.9152 or 0.9522 | M1 | | Allow 3 dp accuracy |
| | minus 0.0789 or 0.1390 | M1 | | Allow 3 dp accuracy |
| | $= 0.836$ | A1 | 3 | AWRT (0.8363) |
| | **or** | | | |
| | B(50, 0.30) expressions stated for **at least 3** terms within $10 \leq B' \leq 20$ | (M1) | | Or implied by a correct answer |
| | Answer = 0.836 | (A2) | | AWRT |
| **(c)** | $n = 40$, $p = 0.7$ | B1 | | Both used; can be implied |
| | Mean $\mu = np = 28$ | B1√ | | CAO; √ on $p$ only |
| | Variance $\sigma^2 = np(1-p) = 8.4$ | M1 | | Use of $np(1-p)$ even if SD |
| | Standard deviation $= \sqrt{8.4}$ or $= 2.89$ to $2.9$ | A1 | 4 | CAO; AWFW |
| | **Total** | | **12** | |

---

| | | | | |
|---|---|---|---|---|
| **6(a)** | Use of binomial in (a) or (b)(i) | M1 | | PI |
| **(i)** | $P(T_{10} \leq 3) = 0.38$ to $0.383$ | B1 | 2 | AWFW (0.3823) |
| **(ii)** | $P(10 < T_{40} < 20) =$ 0.8702 or 0.9256 | M1 | | Allow 3 dp accuracy |
| | minus 0.0352 or 0.0156 | M1 | | Allow 3 dp accuracy |
| | = 0.83 to 0.84 | A1 | | AWFW (0.835) |
| | OR | | | |
| | B(40, 0.40) expressions stated for at least 3 terms within $10 \leq T_{40} \leq 20$ | (M1) | | Or implied by a correct answer |
| | Answer = 0.83 to 0.84 | (A2) | 3 | AWFW |

| | | | | |
|---|---|---|---|---|
| **(b)(i)** | $n = 5 \quad p = 0.4$ | | | |
| | Mean, $\mu = np = 2$ | B1 | | CAO |
| | Variance, $\sigma^2 = np(1-p) = 1.2$ | M1 | | Use of $np(1-p)$ even if SD |
| | Standard deviation $= \sqrt{1.2}$ or $= 1.09$ to $1.1$ | A1 | 3 | CAO<br>AWFW |
| **(ii)** | Mean $(\bar{x}) = 2$ | B1 | | CAO $\sum x = 26$ |
| | Standard Deviation $(s_n,\ s_{n-1})$ $= 1.1$ to $1.16$ | B2 | | $\sum x^2 = 68$<br>AWFW (1.1094 or 1.1547) |
| | If neither correct but use of mean $(\bar{x}) = \frac{\sum x}{13}$ | (M1) | 3 | |
| **(iii)** | Means are same and SDs are similar/same<br>Means are same but SDs are different<br>so<br>Trina's claims appear valid / invalid | B1<br>↑Dep↑<br>B1 | 2 | Must have scored full marks in (b)(i) and (b)(ii) |
| | **Total** | | **13** | |